Test 2C - MTH 1210

Dr. Graham-Squire, Fall 2013

Name: Key

12:57
1:13
→ 16 min.

I pledge that I have neither given nor received any unauthorized assistance on this exam.

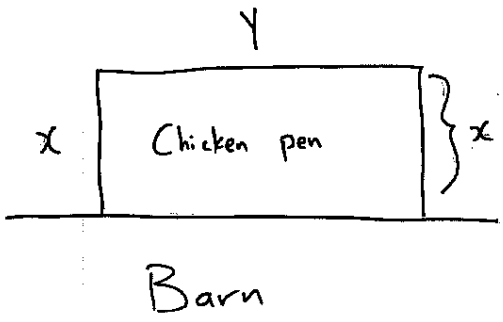
_____

(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones are not allowed on this test. Calculators and/or computers are allowed on the first 3 questions of the test, however you should still show all of your work. No calculators are allowed on the last 6 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 9. Total Points = 85.

1. (10 points) Farmer Bob has decided to build a new rectangular pen to keep his chickens in. The pen will be next to the barn, so he only needs to fence in 3 of the sides. Because he does not want his chickens to think that he is cheap, Bob will make the two sides perpendicular to the barn out of gold fencing (at a cost of \$25 per foot). Let the length of the perpendicular side be x. The side parallel to the barn (with length of y) will be made out of standard fence material, at \$10 per foot. Bob only has \$500 to spend on the fence. Use mathematics to find the maximum possible area for the pen. (Hint: Write an equation for the area of the pen and another equation for the cost of the fencing. Use those to write an expression $A(x)$ for the area of the pen in terms of the single variable x. Then find the maximum of $A(x)$.)



$2x(25) =$ cost for gold fence ✓

$10y =$ cost for standard fence. ✓

$\Rightarrow 50x + 10y = 500$ ✓

$\Rightarrow \frac{10y}{10} = \frac{500 - 50x}{10}$ ✓

$y = 50 - 5x$

Area $= x \cdot y$ ✓✓

$\Rightarrow A(x) = x(50 - 5x)$ ✓

$A(x) = 50x - 5x^2$

Max at $x = \frac{-b}{2a} = \frac{-50}{2(-5)} = 5$ ✓✓

$A(5) = 50(5) - 5(5^2)$

$= 250 - 125$ ✓

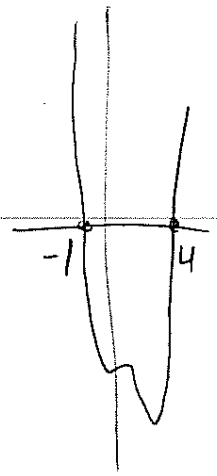
$= \boxed{125 \text{ ft}^2}$

2X. (10 points) Find all zeroes of the polynomial $g(x) = x^4 - 5x^3 + 5x^2 - x - 12$.

No tricks work, so graph to get

$\Rightarrow -1, 4$ are zeroes

$\Rightarrow (x+1), (x-4)$ are factors ✓✓



$\Rightarrow x^2 - 3x - 4$ is a factor

$$x^2 - 3x - 4 \;\overline{)}\; x^4 - 5x^3 + 5x^2 - x - 12$$

$x^2 - 2x + 3$ ✓

$-(x^4 - 3x^3 - 4x^2)$ ✓ ✓

$-2x^3 + 9x^2 - x$

$-(-2x^3 + 6x^2 + 8x)$ ✓

$3x^2 + 9x - 12$

$-(3x^2 - 9x - 12)$

0

**

$\Rightarrow$ Need to factor $x^2 - 2x + 3$ ✓

$\Rightarrow \dfrac{2 \pm \sqrt{4 - 4(1)(3)}}{2}$

$= \dfrac{2 \pm \sqrt{-8}}{2}$ ✓

$= \dfrac{2 \pm i\sqrt{4}\sqrt{2}}{2}$ ✓

$= \dfrac{2 \pm 2i\sqrt{2}}{2}$

$= \boxed{1 \pm i\sqrt{2}}$

Zeroes are $-1, 4, 1+i\sqrt{2}, 1-i\sqrt{2}$

Test C **No Calculator** Name: Key

3. (10 points) (i) Let $f(x) = \dfrac{3x^2 - 12x}{x^2 + x - 6}$. Find the following for the graph of f. If something does not exist, write DNE for your answer and explain why it does not exist.

(a) x-intercept(s): $x=0,\ x=4$

(b) y-intercept(s): $y=0$

(c) Vertical asymptote(s): $x=-3,\ x=2$

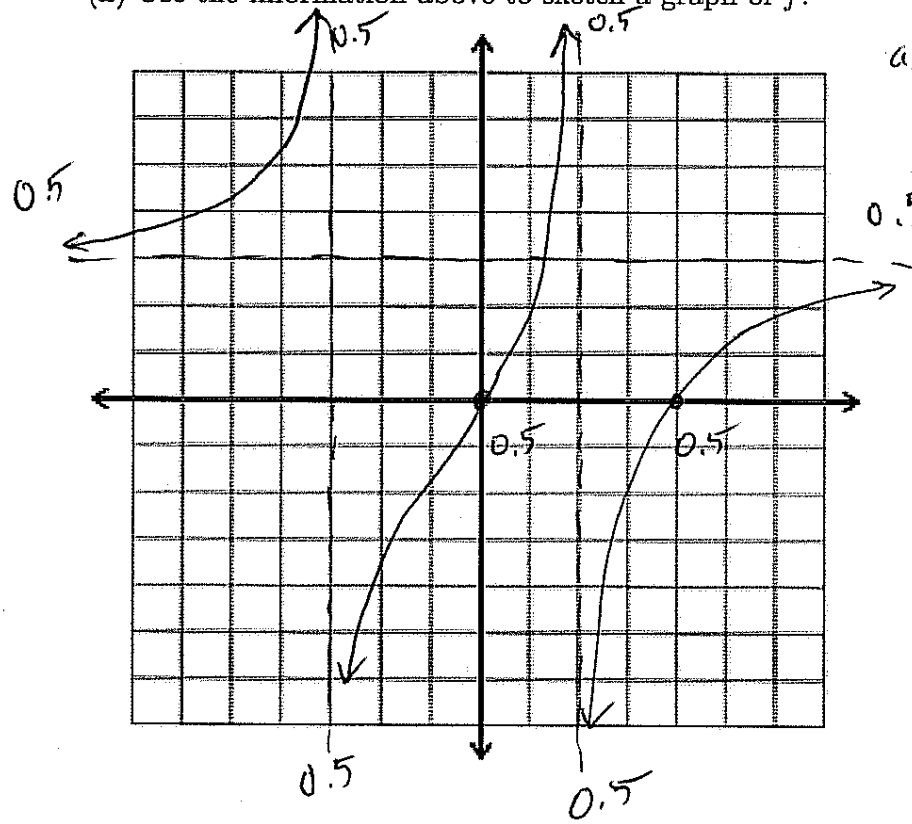
(d) Horizontal asymptote(s): $y=3$

$f(x) = \dfrac{3x(x-4)}{(x+3)(x-2)}$

x-intercepts

vert. asymptotes

(ii) Use the information above to sketch a graph of f.



as $x \to 2^+$, get $f(x) = \dfrac{+\cdot -}{+\cdot +} = -$

as $x \to 2^-$, get $f(x) = \dfrac{+\cdot -}{+\cdot -} = +$

as $x \to (-3)^-$ get $f(x) = \dfrac{(-)(-)}{-\cdot -} = +$

Test A **No Calculator** Name: Key

4. (5 points) In words, describe how you would shift, stretch or flip the graph of $f(x) = \sqrt{x}$ to get the graph of $g(x) = 4\sqrt{-x} - 3$. Make sure to specify in what order you do the changes.

$\sqrt{-x}$ ⇒ first I would flip the graph over the y-axis.

$4\sqrt{-x}$ ⇒ Next I would vertically stretch the graph by a factor of four.

$4\sqrt{-x} - 3$ ⇒ Then I would shift the graph down 3 units.

3 for identifying

2 for order

5. (10 points) Let $f(x) = \sqrt{x}$, $g(x) = \dfrac{x}{x-1}$ and $h(x) = x^4$. Unless otherwise stated, you should simplify the expression given below. Assume that your answer should be a real number. If the expression is undefined, write DNE (does not exist) and explain why it does not exist.

(a) $h(g(f(x)))$ (You do not need to simplify this one)

(b) $h(f(-2))$

(c) $f(h(-2))$

(a) $g(f(x)) = \dfrac{\sqrt{x}}{\sqrt{x}-1}$ ✓✓✓✓

$h(g(f(x))) = \left(\dfrac{\sqrt{x}}{\sqrt{x}-1}\right)^4$

(b) $f(-2) = \sqrt{-2}$ **DNE** b/c can't divide by negative #. ✓✓✓

(c) $h(-2) = (-2)^4 = 16$

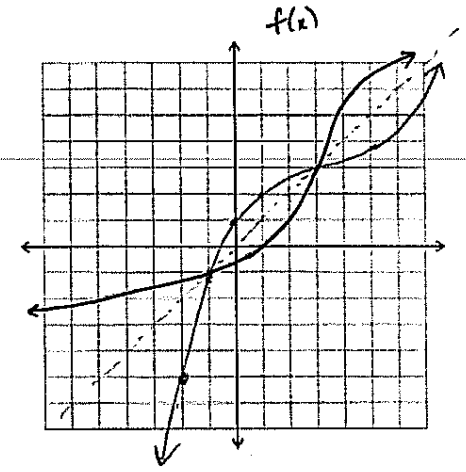
$f(h(-2)) = f(16) = \sqrt{16} = 4$ ✓✓✓

↓ −1 for i

−1.5 for $(\sqrt{-2})^4 = 1$

6. (10 points)

(a) Given the graph of $f(x)$ below, sketch its inverse function $f^{-1}(x)$ on the same set of axes.



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(b) Find the inverse of $g(x) = \dfrac{3x}{x-4}$

✓ $y = \dfrac{3x}{x-4} \Rightarrow x = \dfrac{3y}{y-4}$ ✓

$x(y-4) = 3y$ ✓

$xy - 4x = 3y$ ✓

$xy - 3y = 4x$ ✓

$y(x-3) = 4x$

$y = \dfrac{4x}{x-3} = f^{-1}(x)$ ✓

6

7. (10 points) Factor the polynomial $f(x) = x^4 - 9x^2$, and then use your factorization to graph the function.

$= x^2(x^2-9)$

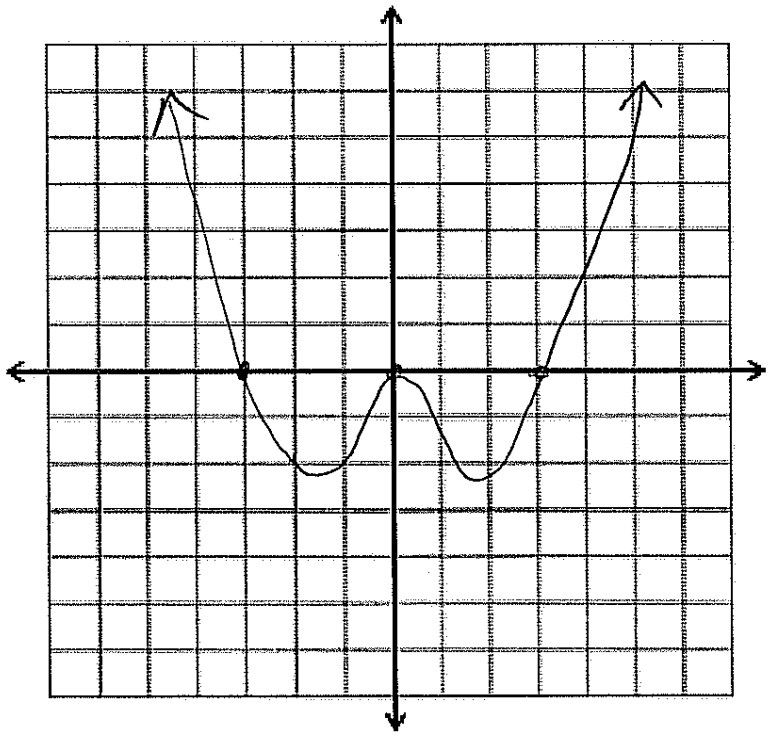
$= x^2(x+3)(x-3)$ ✓✓✓✓

$f(-4) = + \cdot - \cdot - = +$ ✓

$f(-1) = + \cdot + \cdot - = -$ ✓

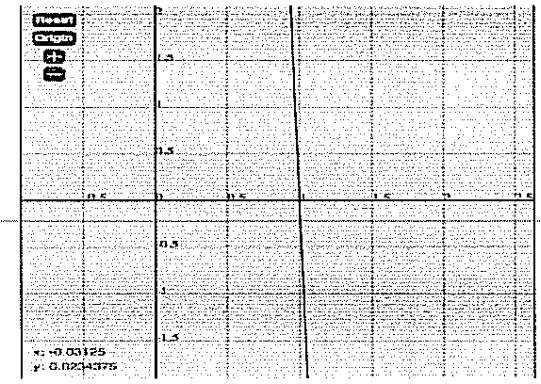
$f(1) = + \cdot + \cdot - = -$ ✓

$f(4) = + \cdot + \cdot + = +$ ✓



✓

8. (10 points) A piece of the graph of $g(x) = x^3 - 8x^2 - 23x + 30$ is given below. Use the graph and polynomial long division to completely factor $g(x)$.



$\Rightarrow 1$ is a zero ✓✓

$\Rightarrow (x-1)$ is a factor.

$$x-1 \;\overline{)\; x^3 - 8x^2 - 23x + 30}$$

$x^2 - 7x - 30$

$-(x^3 - x^2)$ ✓

$-7x^2 - 23x$ ✓

$-(-7x^2 + 7x)$ ✓

$-30x + 30$

$-(-30x + 30)$ ✓

0

$\Rightarrow g(x) = (x-1)(x^2 - 7x - 30)$ ✓✓✓

$= \boxed{(x-1)(x-10)(x+3)}$

9. (10 points) Completely factor the polynomial $f(x) = x^4 - x^3 + 4x^2 - 4x$.

$= x(x^3 - x^2 + 4x - 4)$

$= x(x^2(x-1) + 4(x-1))$

$= x(x^2+4)(x-1)$

$= x(x+2i)(x-2i)(x-1)$

$27 \cdot 0.8 =$

16
56
22

$\Rightarrow$ Need 6 to do 1 point

Extra Credit(up to 2 points) Choose if you want 1 point extra credit or 2 points extra credit. If you choose 1 point, you are guaranteed to get that 1 point. If you choose 2 points and more than 80% of the class also chooses 2 points, then you get zero. If less than 80% of the class chooses 2 points, then you get the full 2 points.

	1 point	2 points
02	9	3
04	4	4